

# UNBOUND

[ WHERE SECURITY IS KEY ]

## Introduction to MPC

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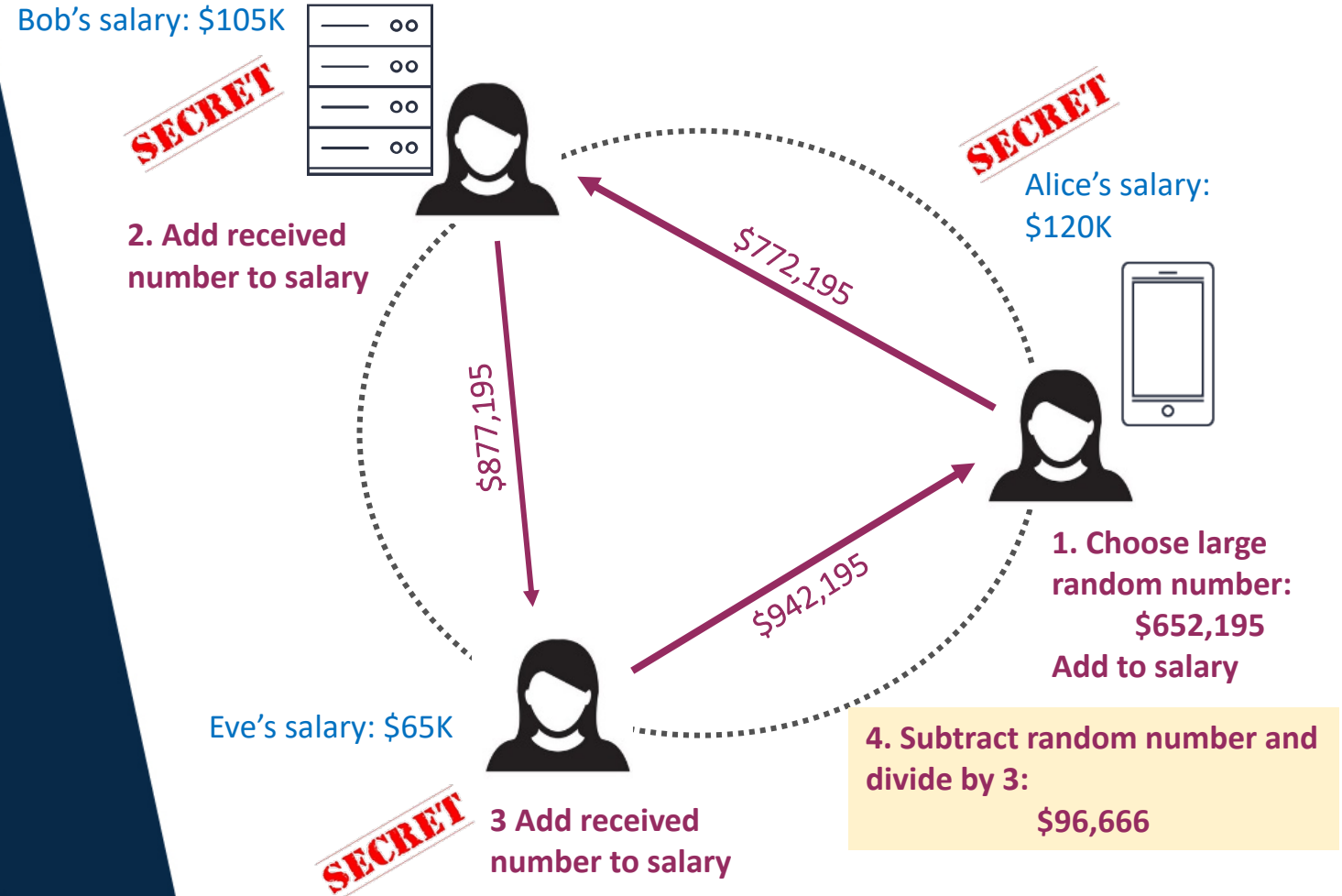
# What is MPC?

# Secure Multiparty Computation

- **A well researched subfield of cryptography**
  - Research began in the late 1980s
  - Thousands of research papers
  - Research was purely theoretical until recently
  - MPC is now a **very active applied area of research**
- **The idea – compute on private data without revealing anything**

# Secure Multiparty Computation Toy Example

**Compute average salary:**  
A group of cryptographers want to compute the average of their salaries, without revealing anyone's salary!



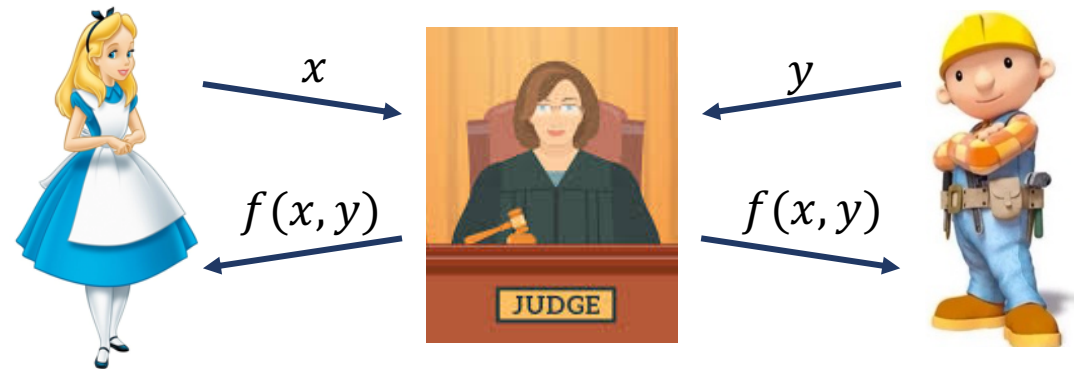


# MPC Security Requirements

- **Parties with private inputs compute a joint function of their inputs**
  - Ensuring that **nothing but the output is learned** (privacy)
  - Ensuring that the **output is correctly computed** (correctness)
- **Properties should be guaranteed even in the face of adversarial behavior**
  - **Semi-honest**: adversary running the correct software cannot learn anything
  - **Malicious**: adversary running **any software** cannot learn anything
    - Even if they know all the protocols, design, and so on
- **The adversary can corrupt parties:**
  - Two main settings: any number (**dishonest majority**) or a minority (**honest majority**)
- **Security is mathematically proven**

# The Ideal/Real Paradigm

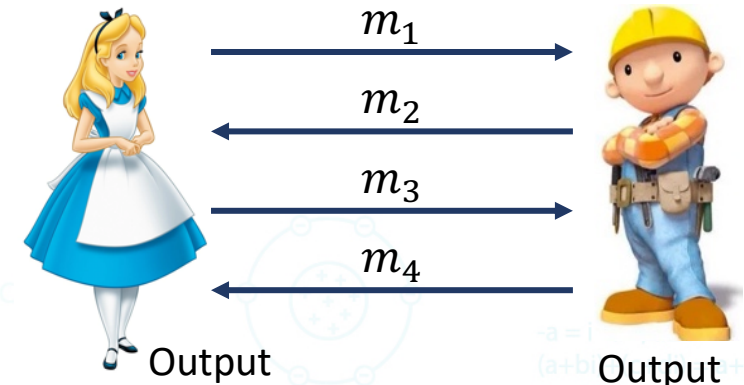
- **What would the ideal situation be?**
  - A trusted and incorruptible third party
  - All parties send inputs to trusted party
    - On perfectly-secure communication channels
  - Trusted party computes and sends output
- **Properties**
  - **Privacy:** each party learns nothing but their output
  - **Correctness:** output is correct
  - **More...**



# The Ideal/Real Paradigm

- **The real world**

- Parties interact with each other
- There is no trusted party
- Parties output what the protocol tells them to



- **Definition: an MPC protocol is secure if it “behaves like” an ideal world protocol**

- Cannot do more than what an attacker can do in the ideal world
- In the ideal world, can choose your input and that's it

# Definitional Advantages

- **Very easy to understand – build and justify your application assuming a trusted party (secure black box)**

- Don't need to be a cryptographer!

- **Words of warning**

- MPC talks about the process but not the function itself
  - Average of salaries between two people reveals everything in the ideal world
  - MPC of cryptographic functions is fine by definition, if cryptographic function is secure
- Parties can choose their own inputs – if this is a problem, needs to be worked into the function definition

# How Does MPC Work?



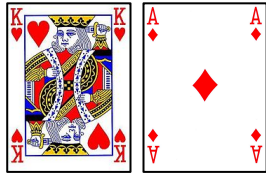
# Secure Computation – A Fun Problem

- **Consider the dating problem**
  - A guy and a girl want to check if they are both interested in going out
  - If they both are, then output is YES
  - If at least one is not, then output is NO
- **If Alice says YES and Bob says NO, then the result is NO and Bob doesn't know if Alice said YES or not**
  - Alice doesn't lose face...

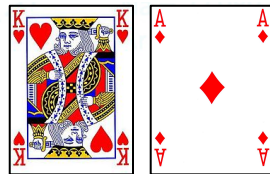


# The Dating Problem with Cards

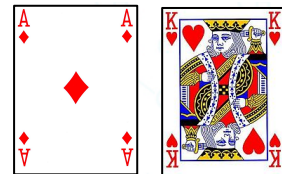
- Alice and Bob each get two cards



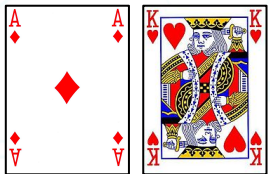
- If Alice likes Bob:



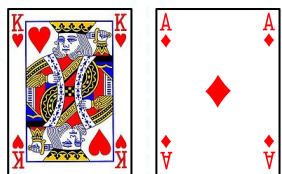
and if not:



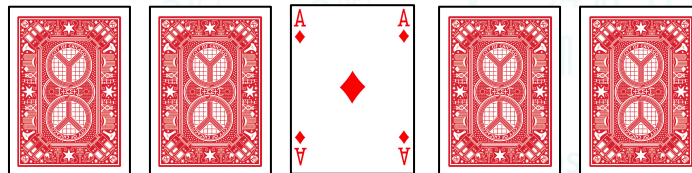
- If Bob likes Alice:



and if not:



- Each turns their cards over, with an Ace in the middle



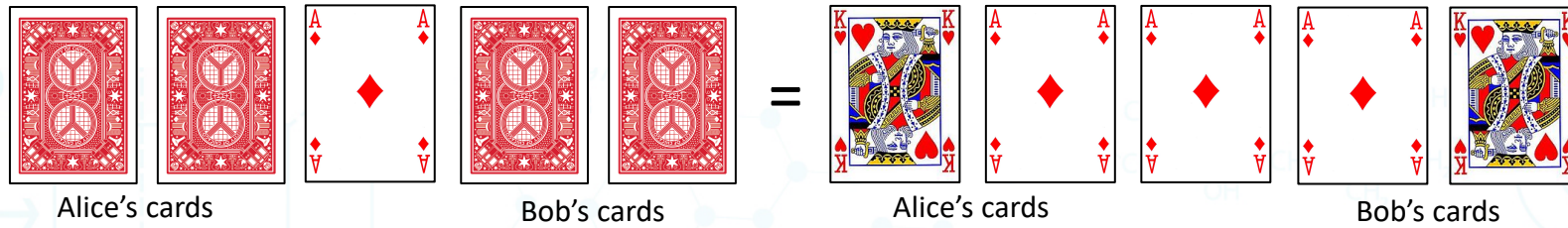
Alice's cards

Bob's cards

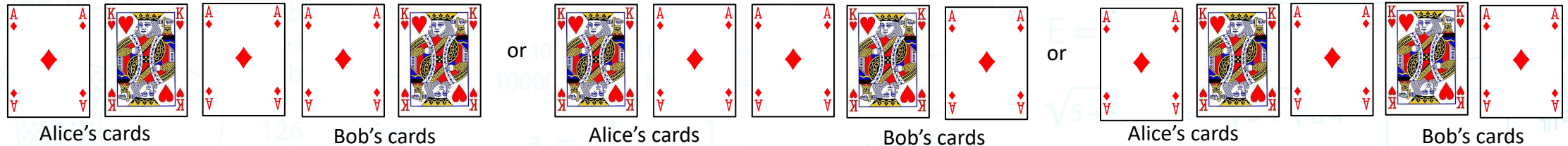


# The Dating Problem with Cards

- If Alice and Bob like each other



- Otherwise,



- Parties turn over middle card and randomly rotate
- If three Aces in a row then YES; else NO

# General Secure Computation

- **Powerful feasibility theorems for MPC**

- Any function can be securely computed!

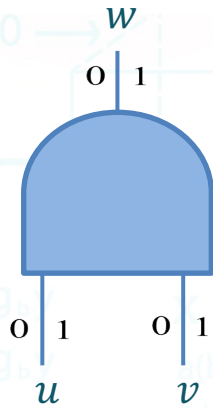
- **How is it possible to securely compute **any function**?**

- Represent the function as a (Boolean or arithmetic) circuit
  - Show how to compute any circuit in MPC

- **Is this even remotely efficient???**

# Yao's Garbled Circuits

- Garbling a single Boolean gate



$u$	$v$	$w = u \wedge v$
0	0	0
0	1	0
1	0	0
1	1	1

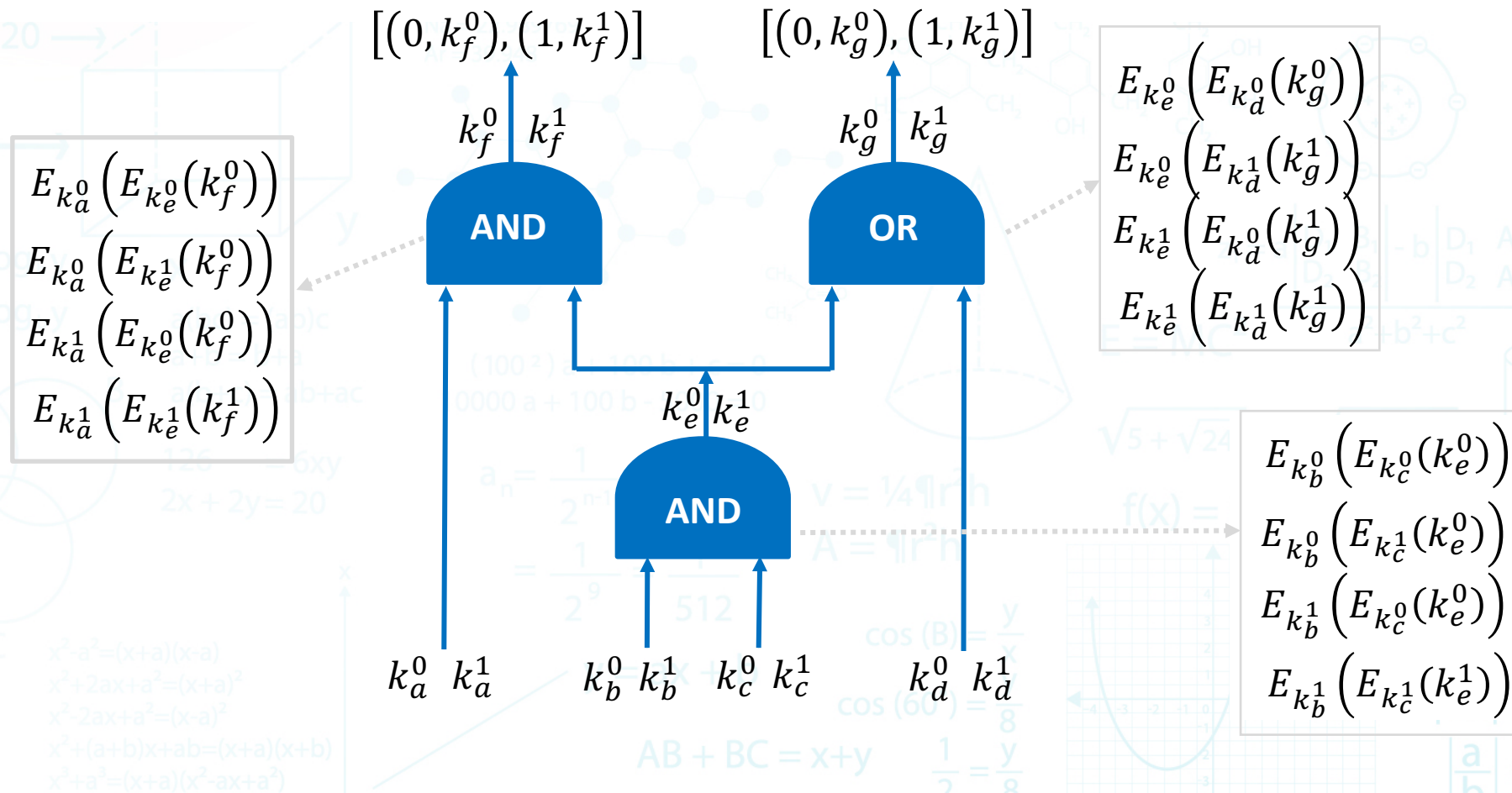
$u$	$v$	$w = u \wedge v$
$k_u^0$	$k_v^0$	$k_w^0$
$k_u^0$	$k_v^1$	$k_w^0$
$k_u^1$	$k_v^0$	$k_w^0$
$k_u^1$	$k_v^1$	$k_w^1$

$$\begin{aligned} &E_{k_u^0}(E_{k_v^0}(k_w^0)) \\ &E_{k_u^0}(E_{k_v^1}(k_w^0)) \\ &E_{k_u^1}(E_{k_v^0}(k_w^0)) \\ &E_{k_u^1}(E_{k_v^1}(k_w^1)) \end{aligned}$$

In random order

- Given one key on each input wire, can compute the key on the output wire, without learning **anything** about the represented values
- Keys on input wires are called **garbled inputs**

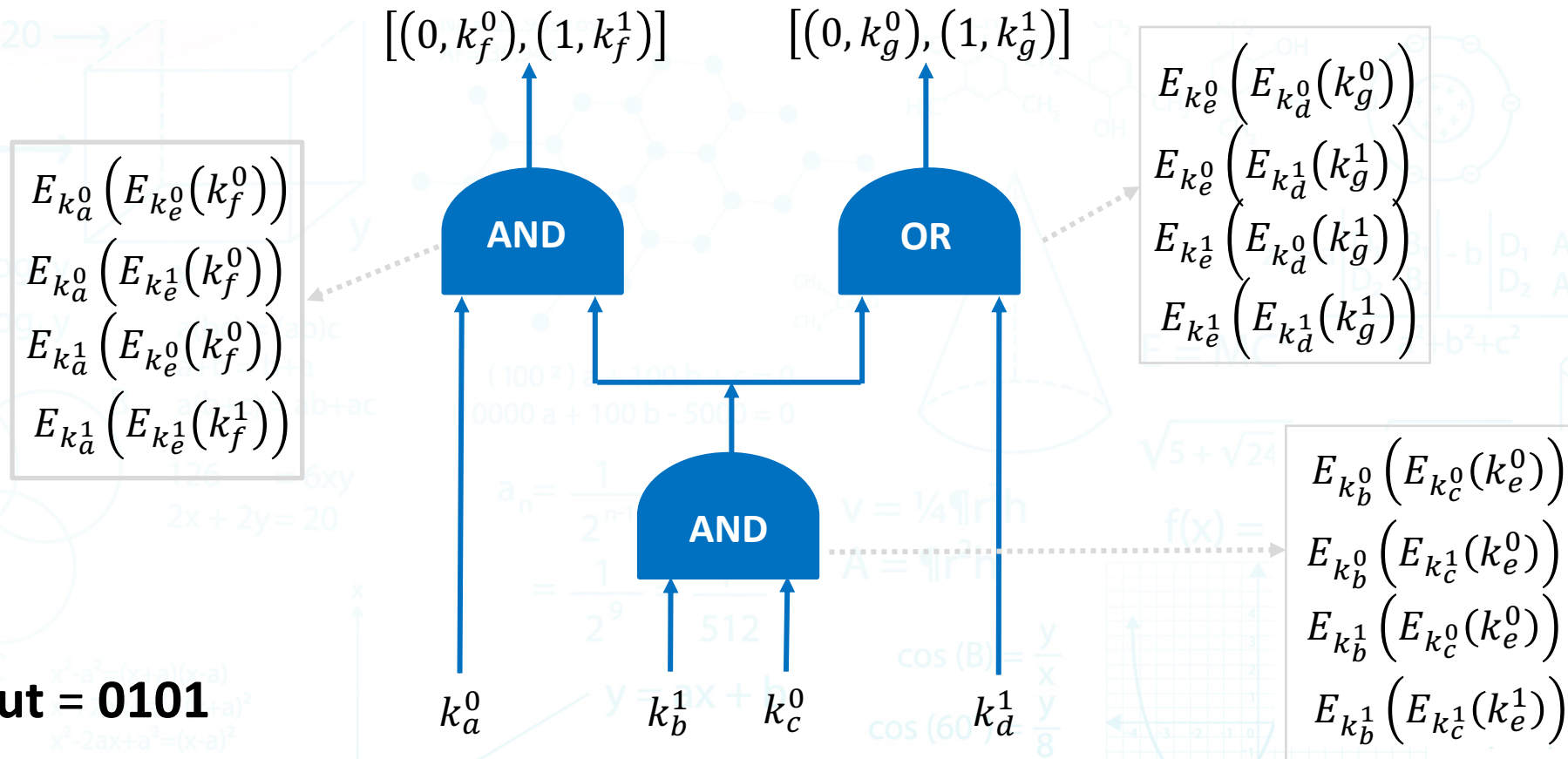
# Garbling an Entire Circuit





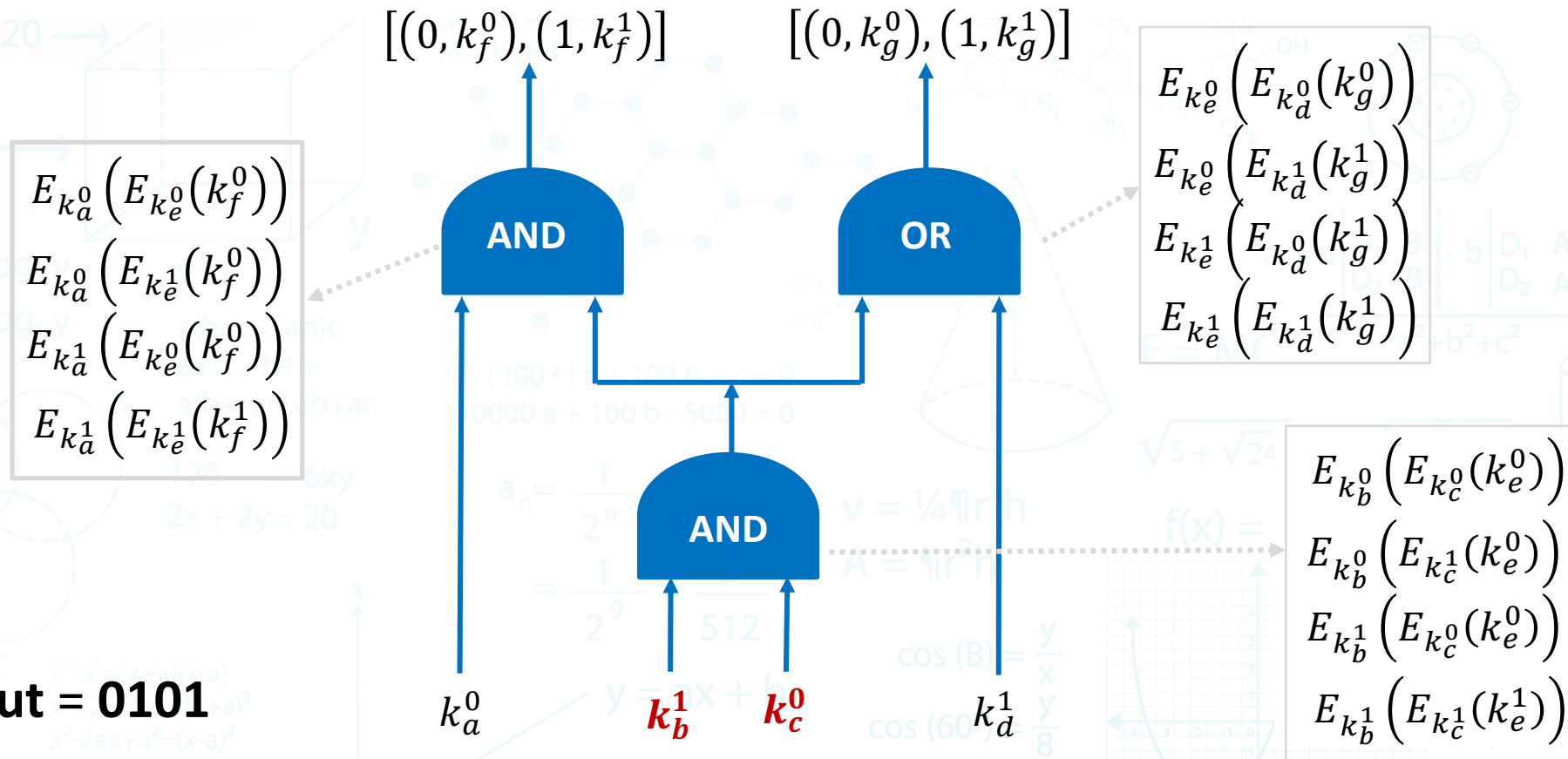
# Computing a Garbled Circuit

Input = 0101

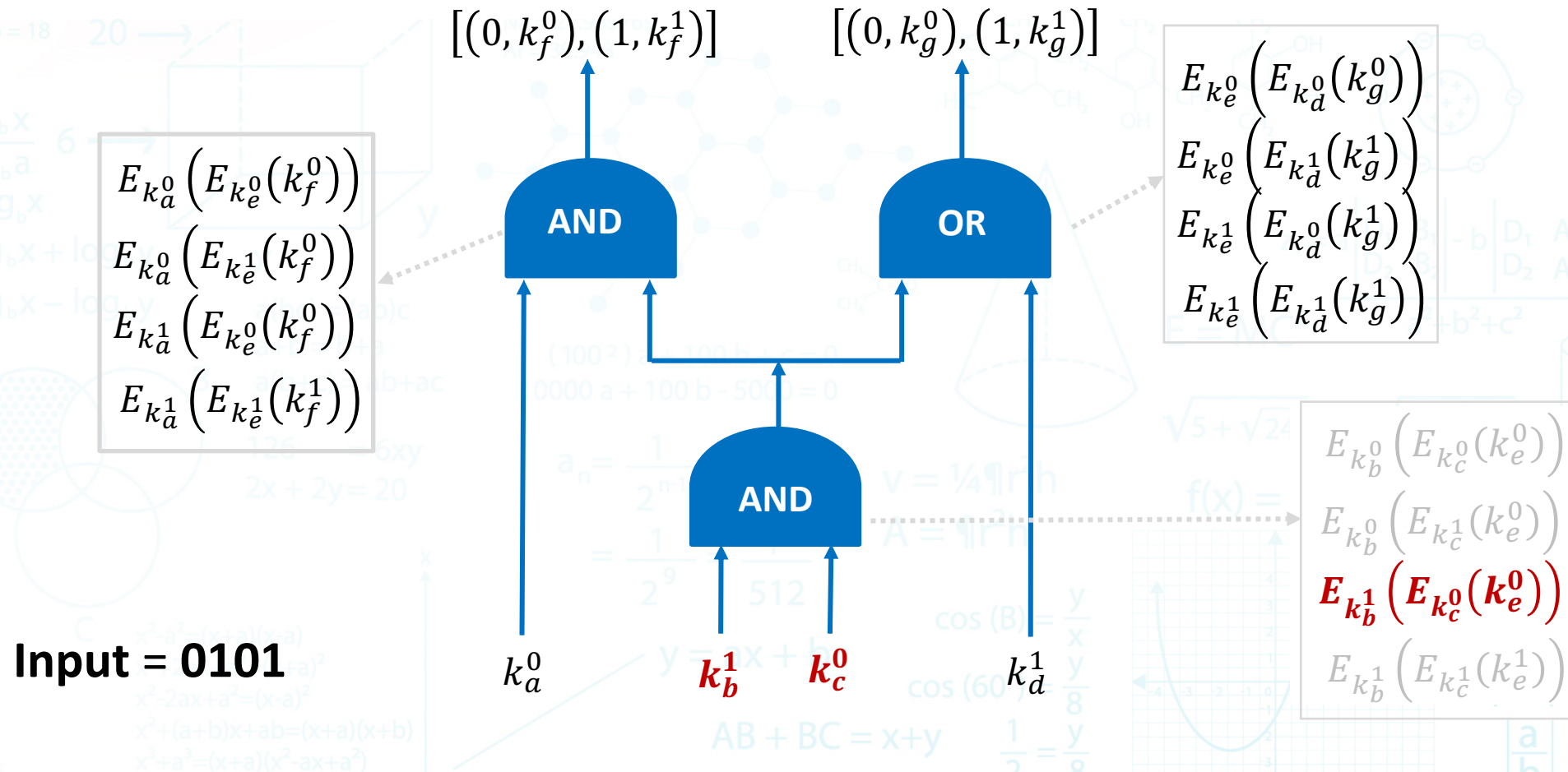


- Computing the Garbled Circuit

Input = 0101

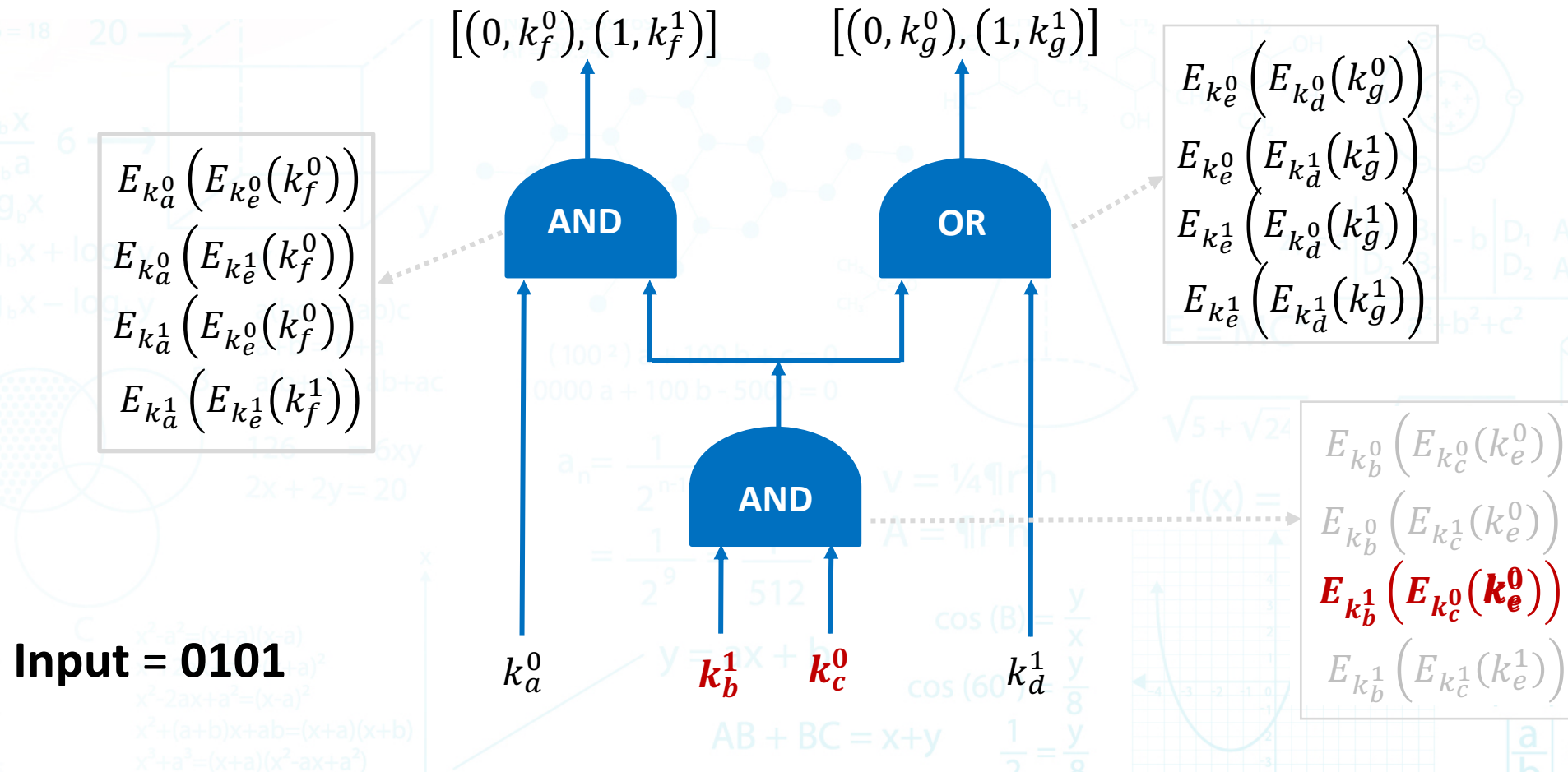


# Computing a Garbled Circuit

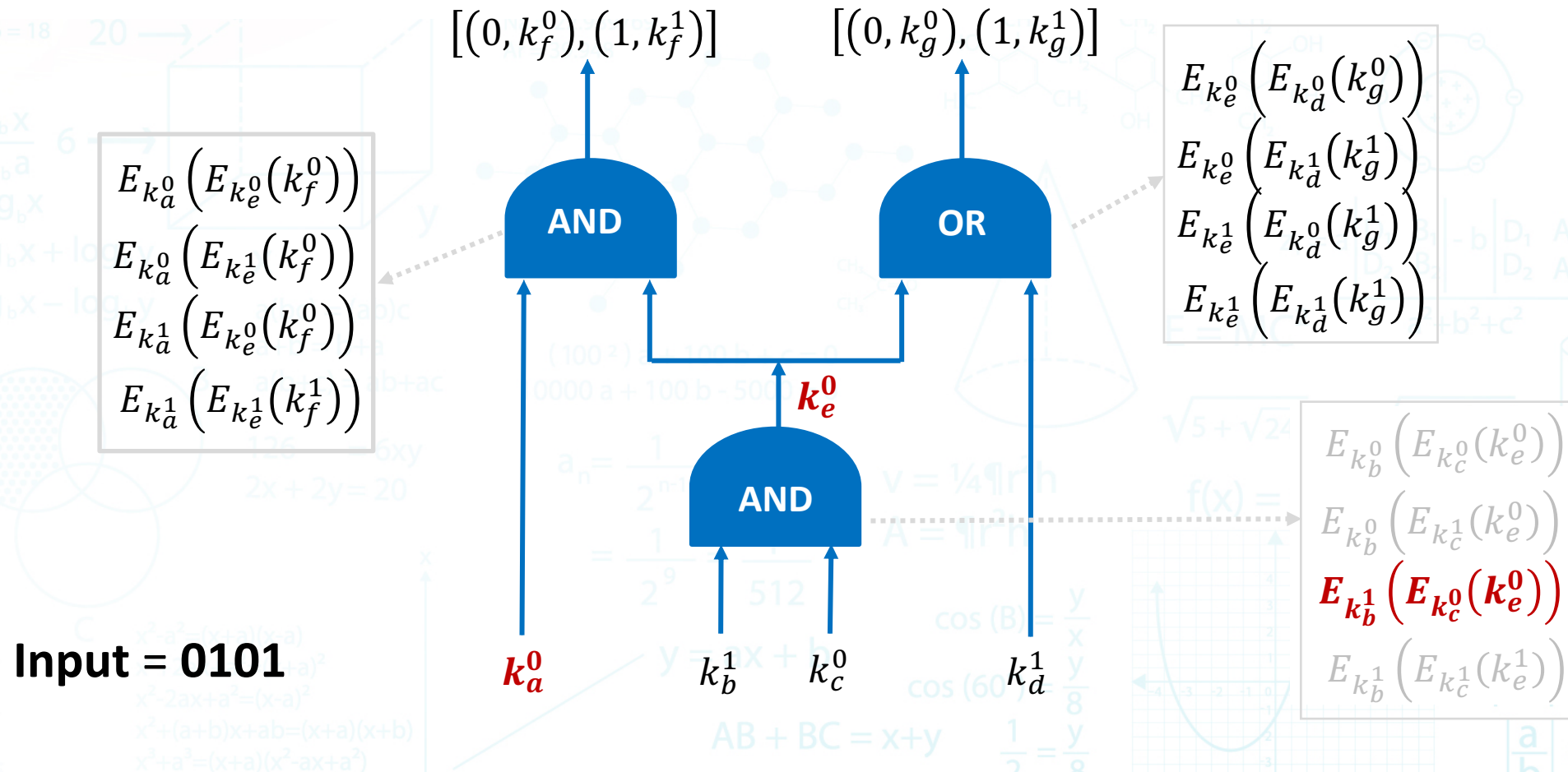




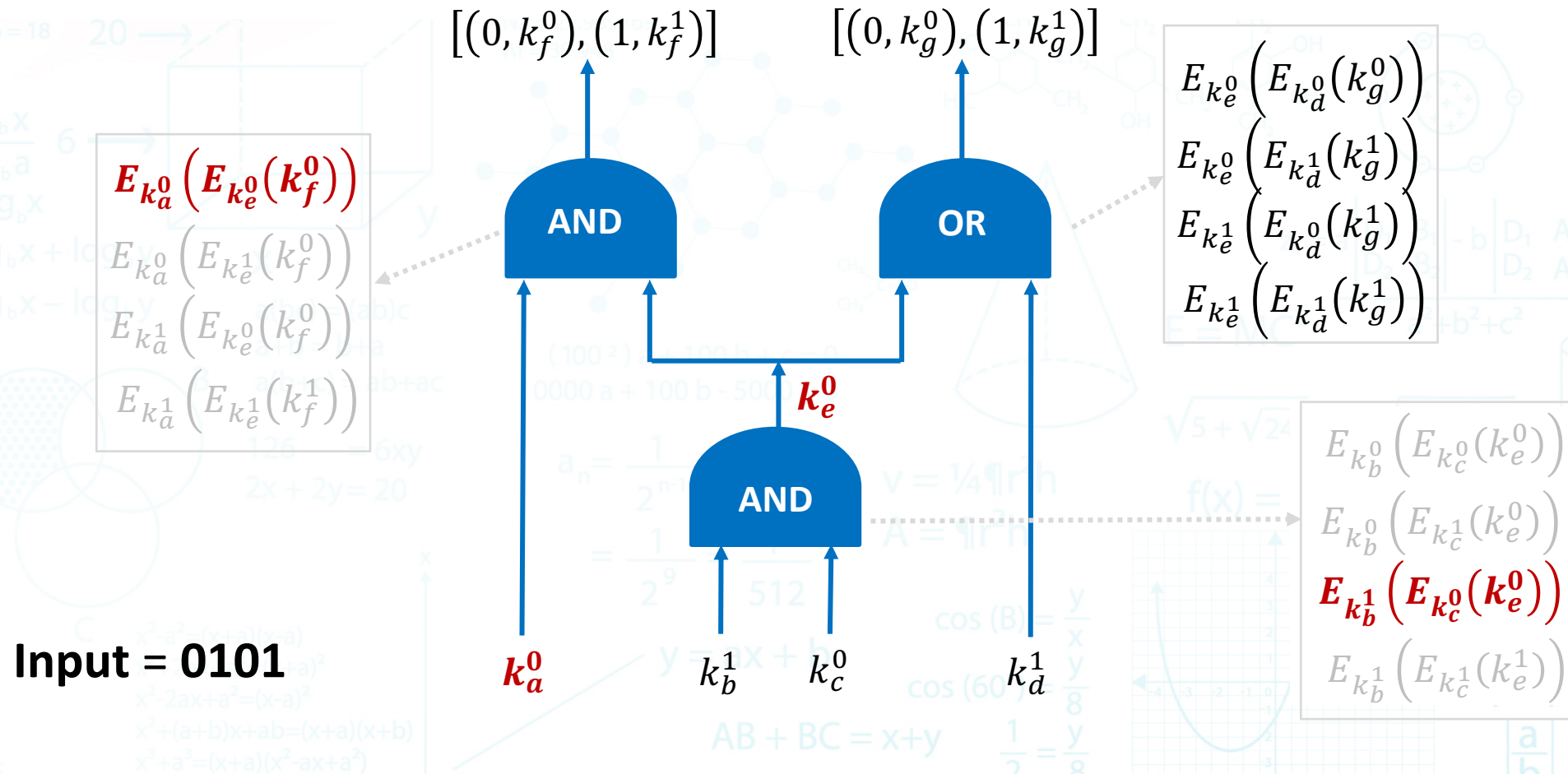
# Computing a Garbled Circuit



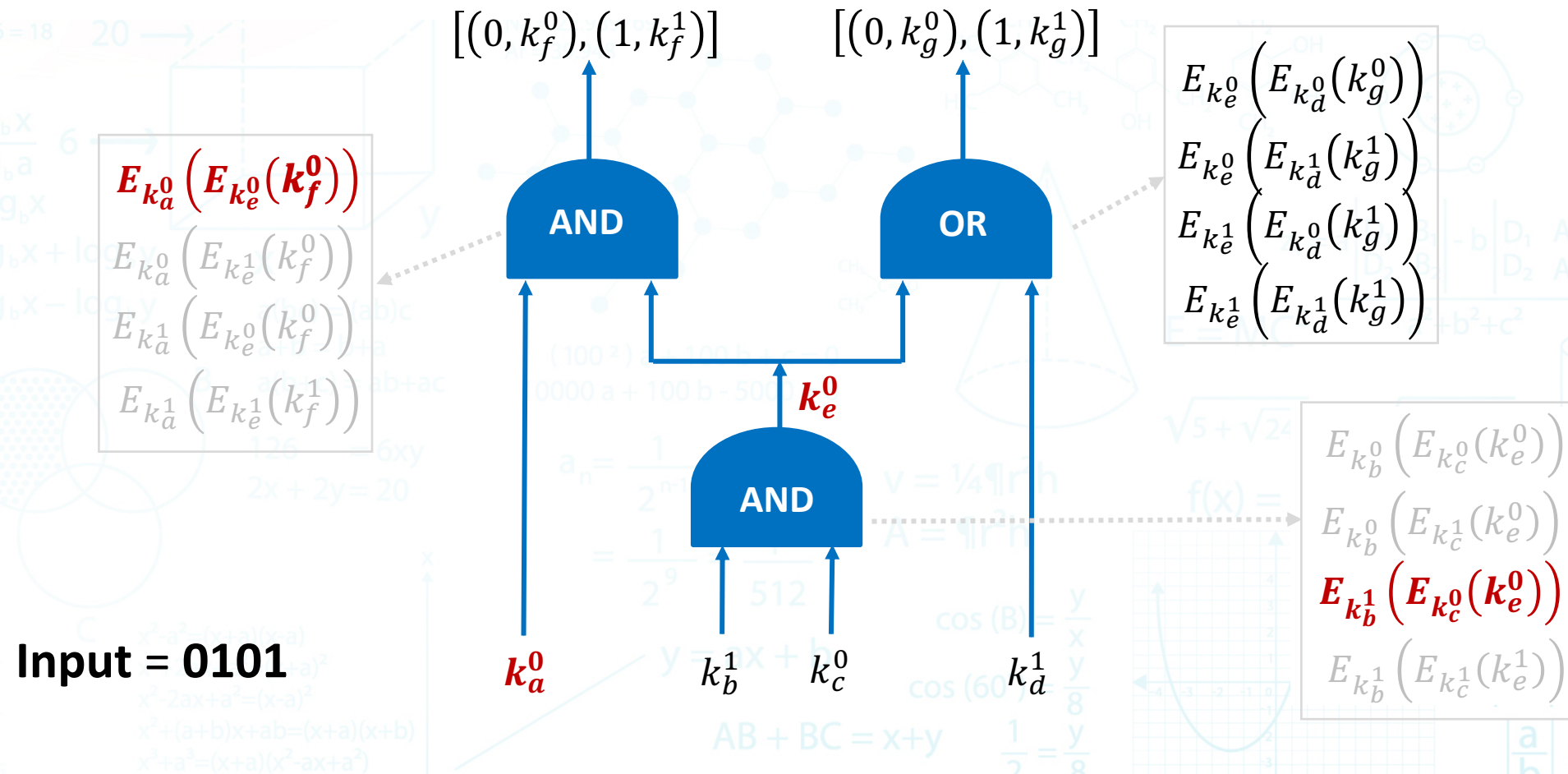
# Computing a Garbled Circuit



# Computing a Garbled Circuit

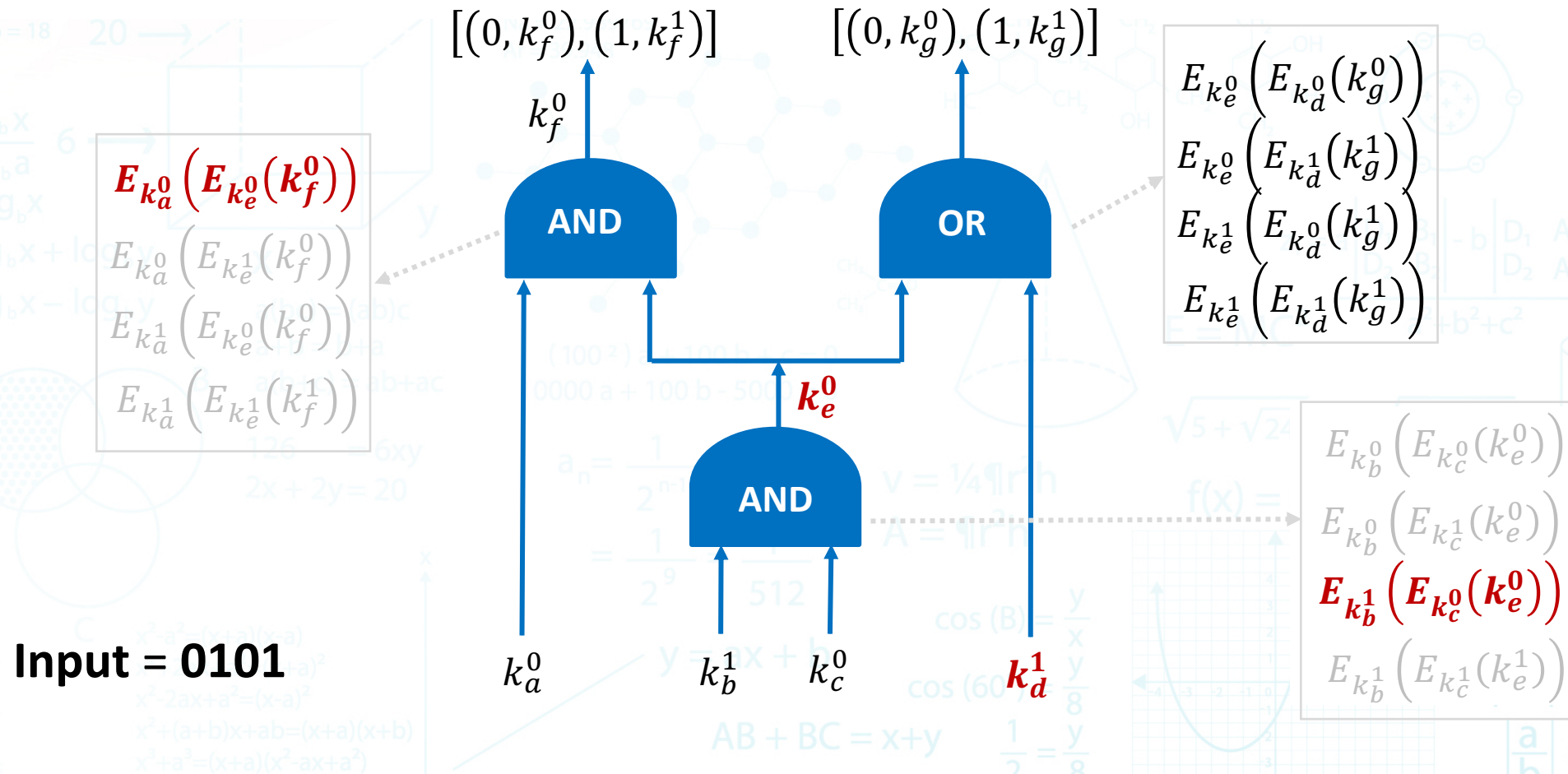


# Computing a Garbled Circuit

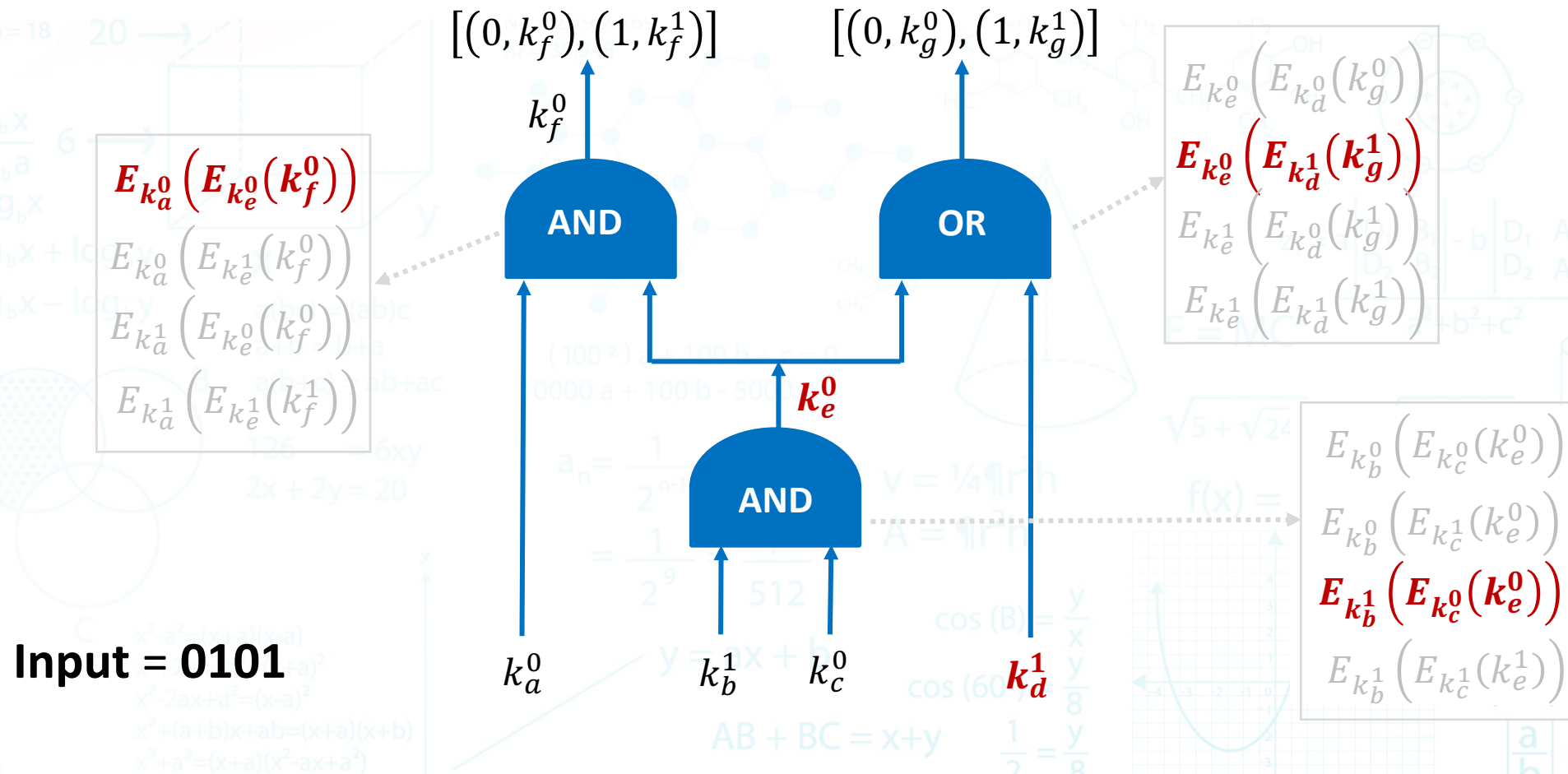




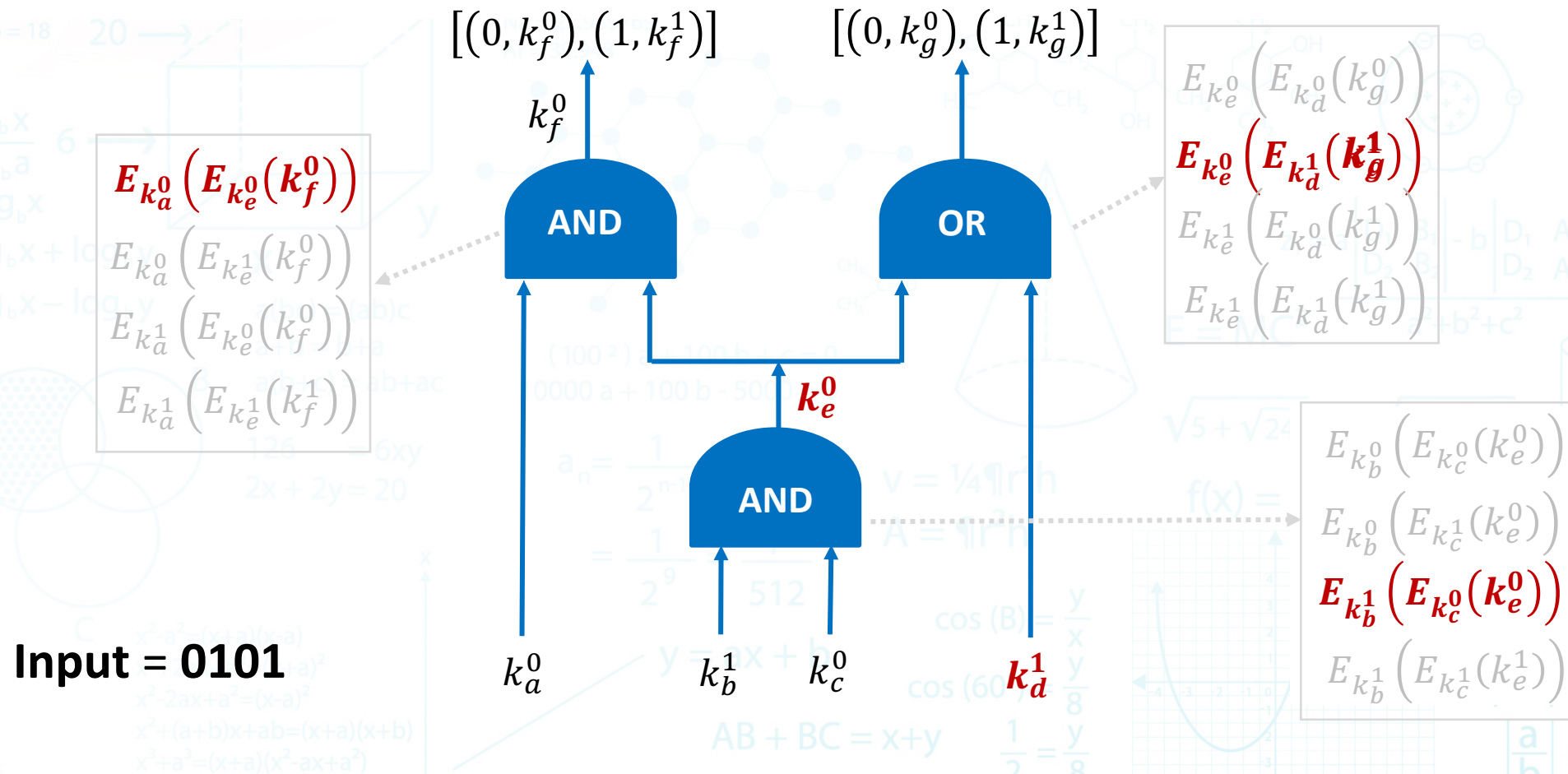
# Computing a Garbled Circuit



# Computing a Garbled Circuit

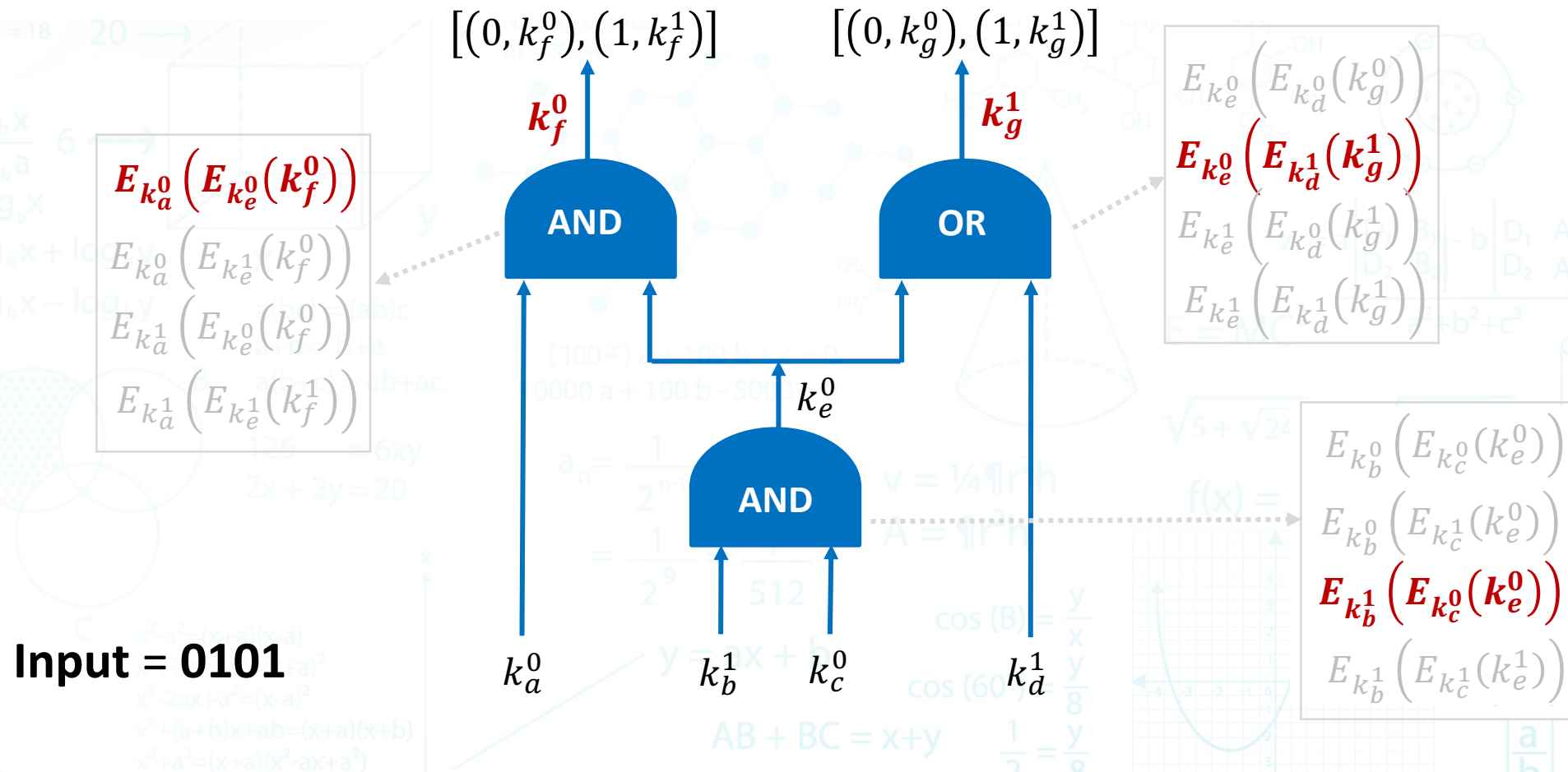


# Computing a Garbled Circuit

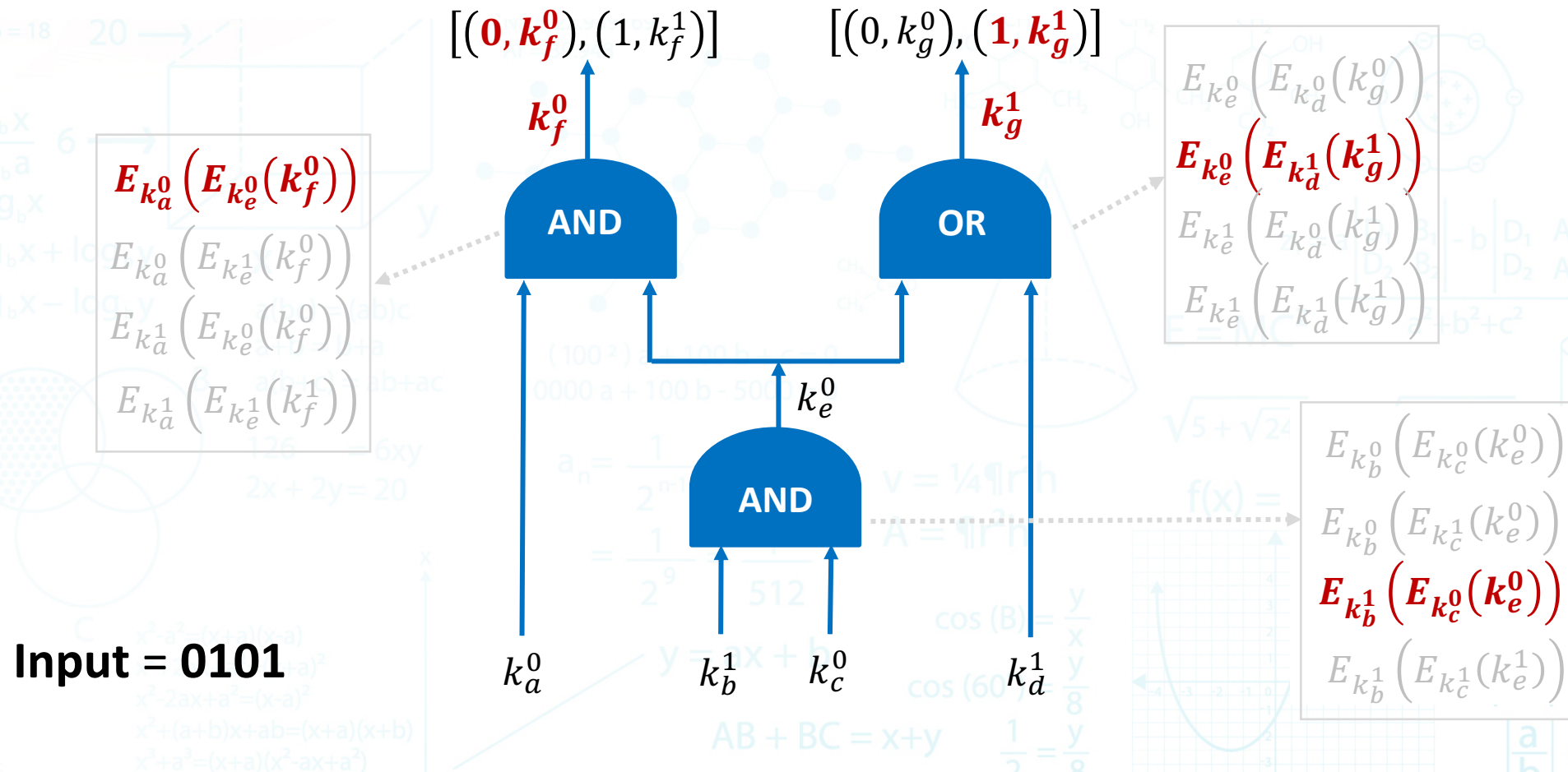




# Computing a Garbled Circuit



# Computing a Garbled Circuit



# Three-Party Protocol with Garbled Circuits

- **Inputs:** party  $P_1$  has input  $x$ , and party  $P_2$  has input  $y$ ; party  $P_3$  has no input
- Protocol with security against one malicious party

- Generate GC from seed
- Generate garbled input of  $x$  to garbled circuit



seed for generating GC



- Generate GC from seed
- Generate garbled input of  $y$  to garbled circuit

GC + garbled input of  $x$

GC + garbled input of  $y$



$f(x, y)$

**There exist protocols for two parties with security against one malicious that use garbled circuits (and also multiparty with dishonest majority)**

- Verify that both garbled circuits are same
- Compute GC on keys and **get output**
- Send output back (authenticated)

# MPC for Specific Tasks

# Threshold Cryptography

- **Compute a cryptographic function without any single party holding the key**
- **Motivation:**
  - Make it hard to steal the key
  - Provide quorum authorizations (like signees for bank transactions)



# Securely Computing the RSA Function

- **RSA signing and decryption:**
  - Private key:  $(d, N)$
  - Public key:  $(e, N)$
  - Private operation (sign/decrypt):  $z = y^d \bmod N$
- **RSA key sharing**
  - Server  $S_1$  has a random  $d_1$
  - Server  $S_2$  has  $d_2 = d - d_1 \bmod \phi(N)$
  - Note that  $d_1 + d_2 = d \bmod \phi(N)$
  - Security:
    - $d_1$  reveals nothing about  $d$  since it's random
    - $d_2$  reveals nothing about  $d$  since  $d_1$  completely hides  $d$

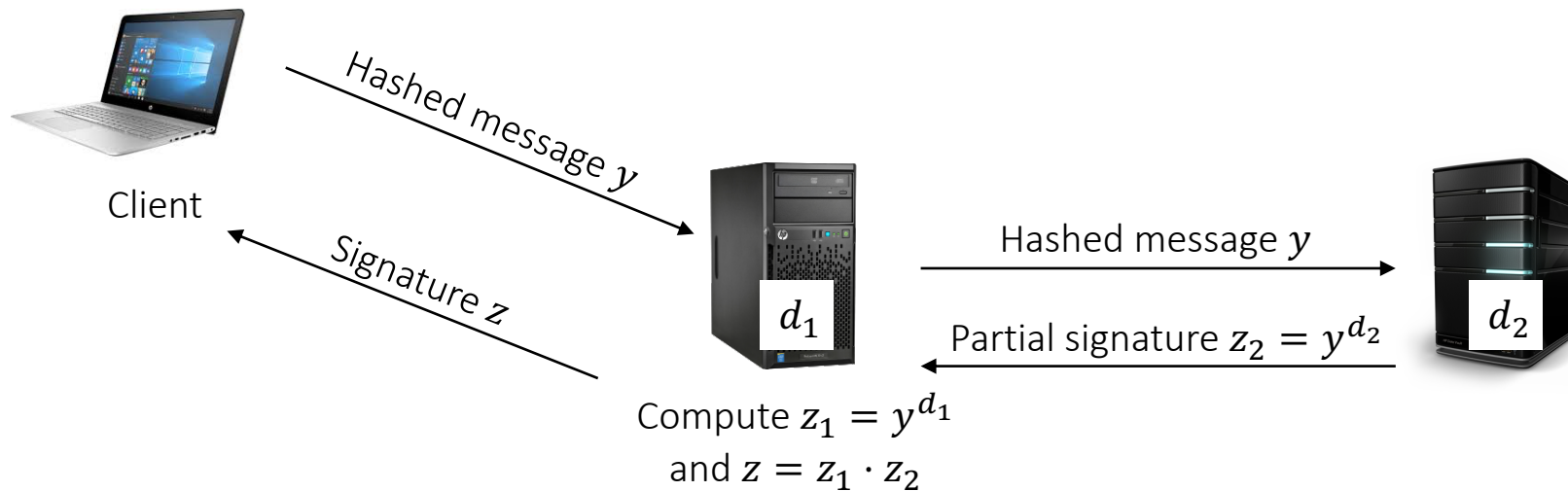
# Securely Computing the RSA Function

- **Recall: server  $S_1$  has  $d_1$  and server  $S_2$  has  $d_2$ , such that  $d_1 + d_2 = d$**
- **Securely computing the private operation  $x = y^d \bmod N$** 
  - Server  $S_2$  computes  $z_2 = y^{d_2} \bmod N$  and sends to server  $S_1$
  - Server  $S_1$  computes  $z_1 = y^{d_1} \bmod N$
  - Server  $S_1$  computes  $z = z_1 \cdot z_2 \bmod N$
  - Server  $S_1$  *verifies the result* by checking that  $y = z^e \bmod N$
  - Note:  $z = z_1 \cdot z_2 = y^{d_1} \cdot y^{d_2} = y^{d_1+d_2} = y^d \bmod N$ 
    - The last equality holds since addition in exponent is mod  $\phi(N)$



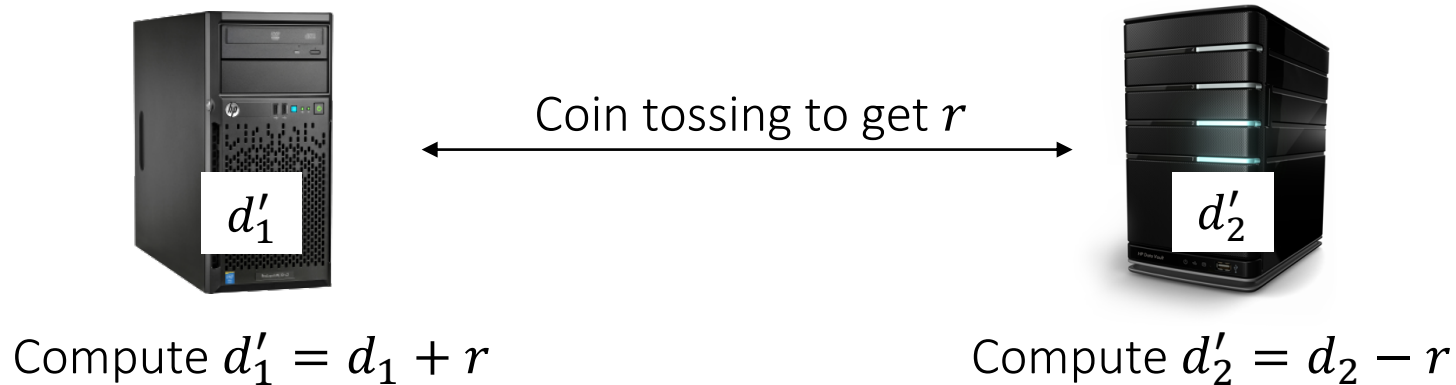
# Two-Party Protocol for RSA Decrypt/Sign

- Inputs: Server  $S_1$  has key share  $d_1$  and hash of message  $y$ , and server  $S_2$  has key share  $d_2$



# Secret Share Refresh – Proactive Security

- At fixed intervals (e.g., every hour), sharing of secret is refreshed
- For RSA:



- Note that given  $d_1$  and  $d'_2 = d_2 - r$ , nothing can be learned about  $d$

# Other Threshold Cryptography

- **As with RSA, it is possible to efficiently compute ECDH, ECDSA, etc.**
  - They all have (mostly) nice algebraic structure
- **What about AES, HMAC, and so on?**
  - As above, convert the function description to a Boolean circuit (AND/XOR gates)
  - Use garbled circuits, or another method
- **Efficiency**
  - AES circuit has about 31,000 gates: 6400 AND and 25000 XOR (but XOR is free)
  - It takes about ½ ms to garble and evaluate an AES circuit
  - We do about 500 AES-256-GCM operations on 32-byte input (key wrap) per second
    - 4-core machine, 10Gbps network

# Private Set Intersection

- **The problem:**
  - Input: Alice has a set  $A$ , Bob has a set  $B$
  - Output: the set  $A \cap B$
- **This problem has many solutions; we will see a conceptually simple one here (for semi-honest adversaries)**
- **Tool – oblivious pseudorandom function evaluation**
  - Input: Alice has a key  $K$ , Bob has an input  $b$
  - Output: Alice learns nothing, Bob learns  $F_K(b)$
  - Concretely, this could be  $AES_K(b)$  or a PRF based on elliptic curves

# Protocol for Private Set Intersection

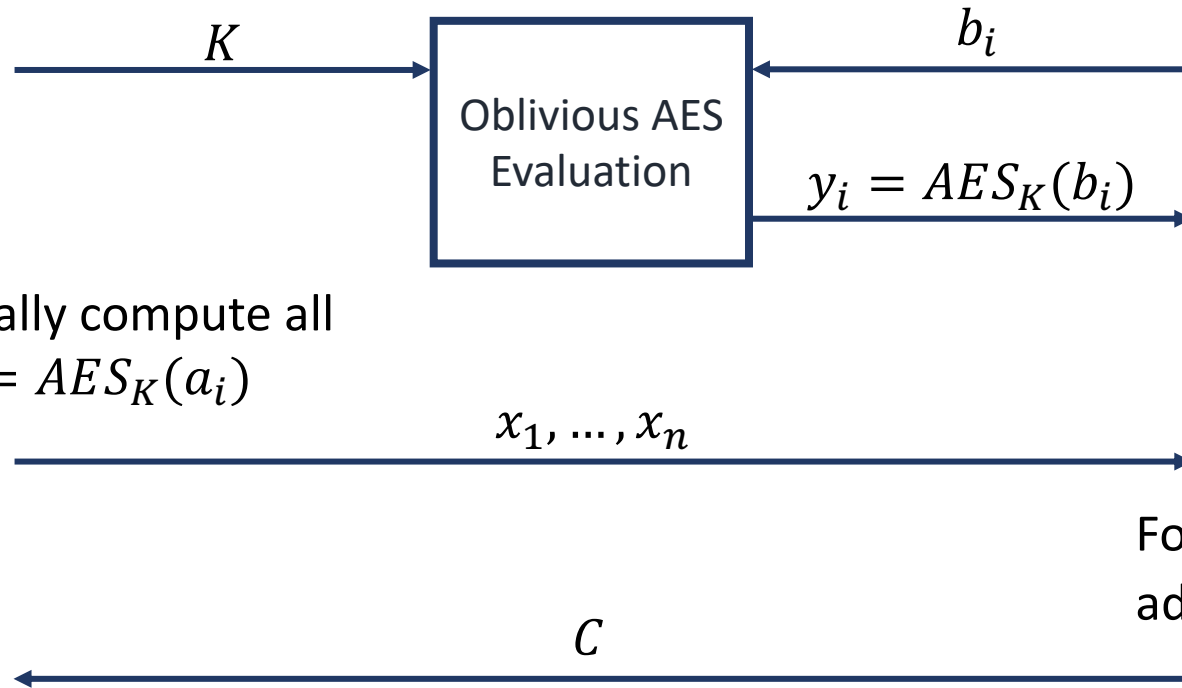
Input:  $A = a_1, \dots, a_n$



Choose random  $K$

Locally compute all  
 $x_i = AES_K(a_i)$

Output  $C$



Input:  $B = b_1, \dots, b_m$



For every  $i$ , if  $y_i = x_j$  for some  $j$ ,  
add  $b_i$  to the output set  $C$

Output  $C$



# Use Cases in Practice

# Advertising Conversion – Google

- **The problem:**
  - How can we determine the effectiveness of advertisements for BMWs shown on someone's cellphone?
- **The solution:**
  - Compute how many people were shown the ad on their cellphone
  - Compute how many people who were shown the ad that bought a BMW
  - (Normalize by expected percentage purchase if not shown the ad)
- **Privacy concern: this requires Google and BMW sharing their lists**
- **Solution: use private set intersection**
  - In fact, it suffices to compute the cardinality (or the sum of amount spent)

# Boston Wage Gap Study

In 2015, women in Boston earned

77 ¢

Data Snapshot:

- 69 employers
- 113,000 FT employees
- \$11 Billion in annual wages

In 2016, women in Boston earned

76 ¢

Data Snapshot:

- 114 employers
- 167,000 FT employees
- \$14 Billion in annual wages

In 2018, women in Boston earned

70 ¢

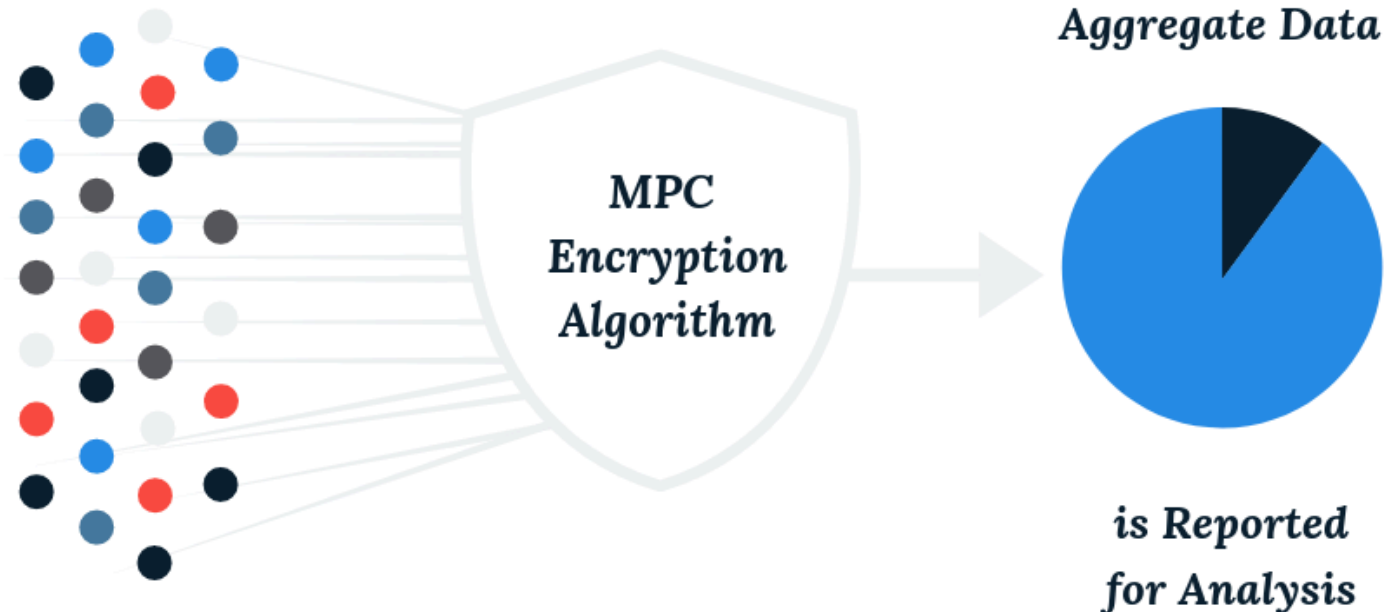
Data Snapshot:

- 125 employers
- 140,000 FT employees
- \$12.2 Billion in annual Wages

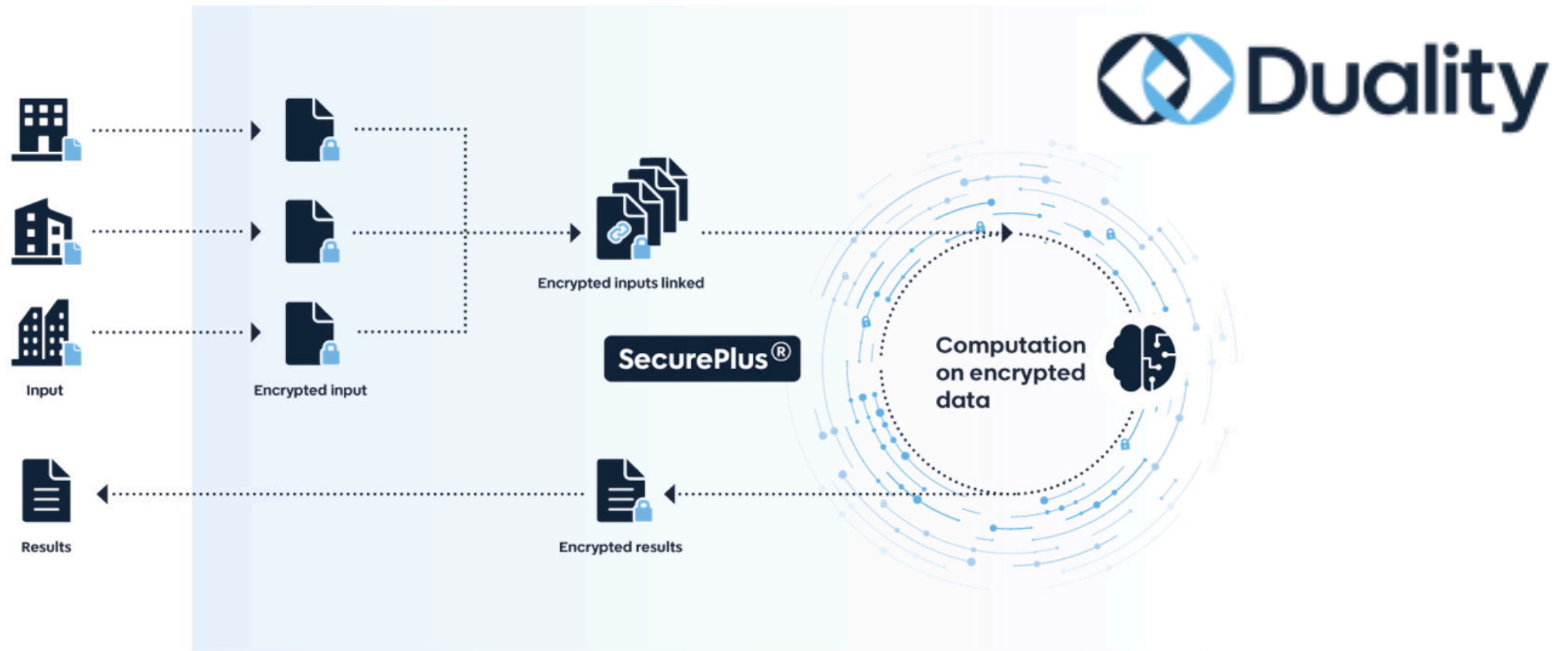
# Boston Wage Gap Study

## How Does it Work?

Using MPC-backed software, 100% Talent Compact members anonymously provide encrypted data on full-time employees, similar to EEOC-1 job designations. The BWWC then accesses the aggregated data to perform analyses on wage gap by gender, race and ethnicity, and new this year - by industry. **The more participants, the more secure and accurate the data is.**



# Privacy-Preserving Analytics/Statistics

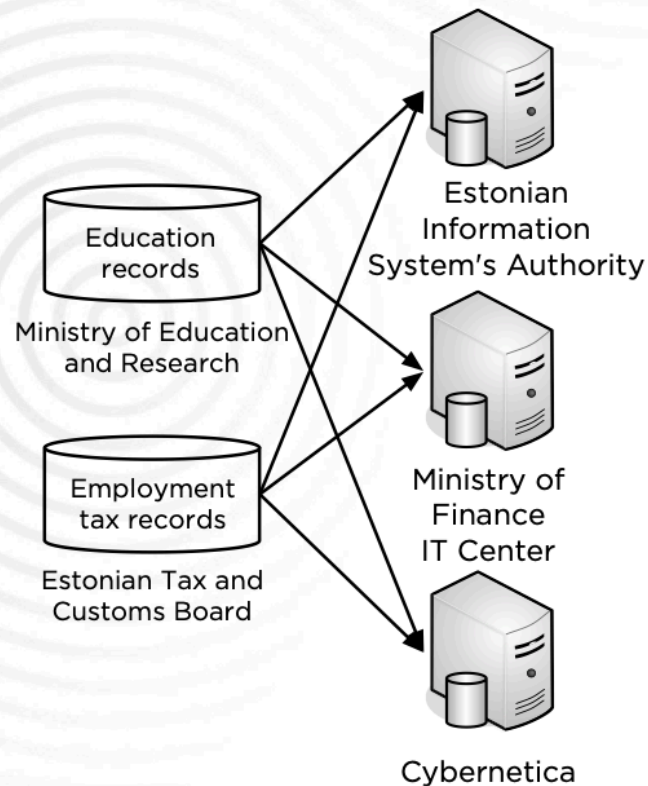


Enabling privacy-preserving statistical analyses across entities and jurisdictions, Duality SecurePlus Statistics® opens up new opportunities for **organizations in regulated industries** such as healthcare, financial services, insurance, retail and telecommunications to collaborate with partners on sensitive data so they can grow their business, improve their research, and drive operational efficiencies.



# Privacy-Preserving Analytics/Statistics

## Sharemind® Powered the Privacy-Preserving Study in the PRIST project



- ⊙ Source data:
  - ⊙ 10 million tax records,
  - ⊙ 600 000 education records.
- ⊙ Sharemind hosted by government agencies and Cybernetica.
- ⊙ Data owners used the Sharemind encryption tools to upload data.
- ⊙ Data never existed outside the source in an unencrypted state.

# MPC for Cryptographic Key Protection

- **Classical MPC use cases consider different parties collaborating**
  - Unbound considers where it all belongs to you, but you don't trust your network

1



Each private key is split into random shares stored on separate locations and continually refreshed

2



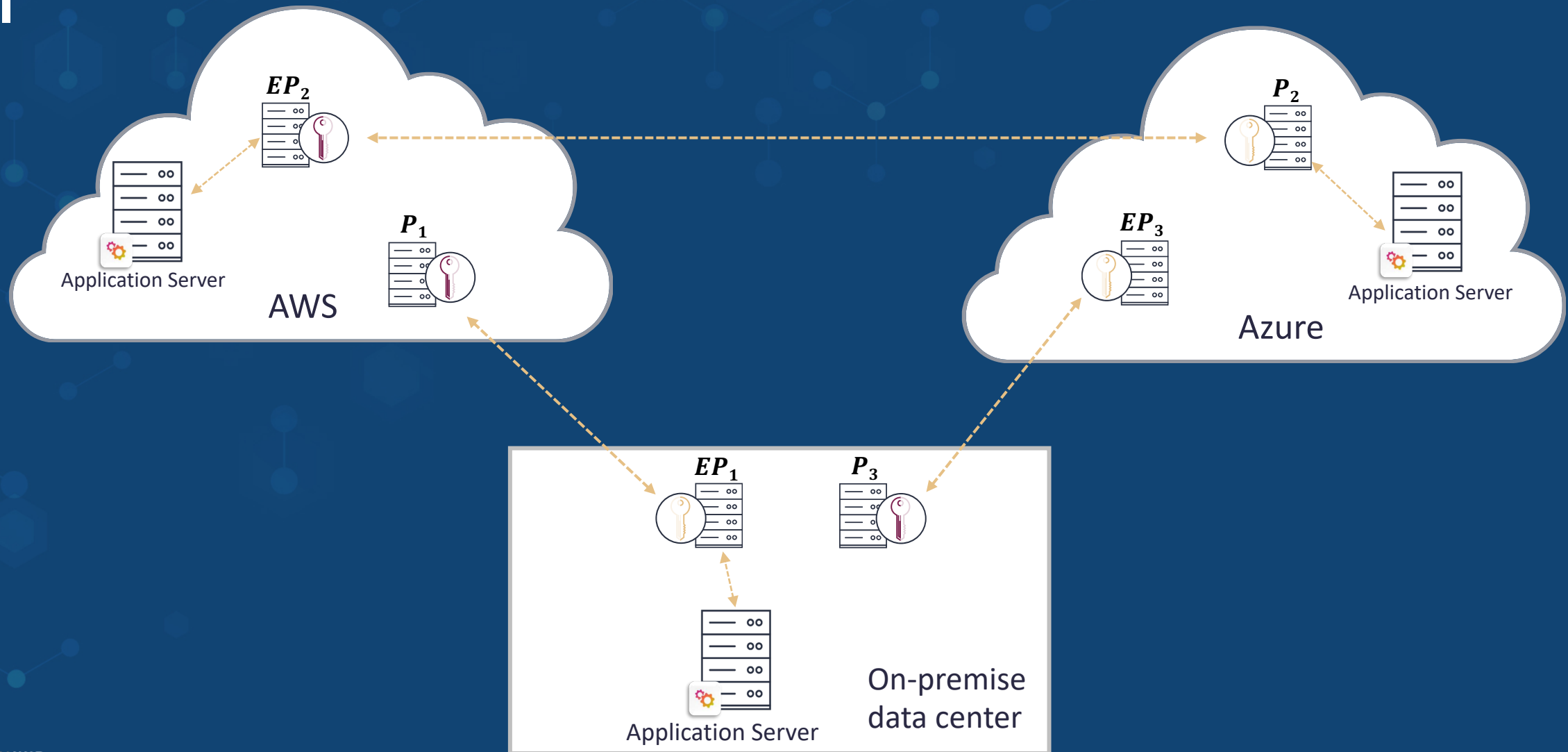
Key shares are never united, from generation through usage

3



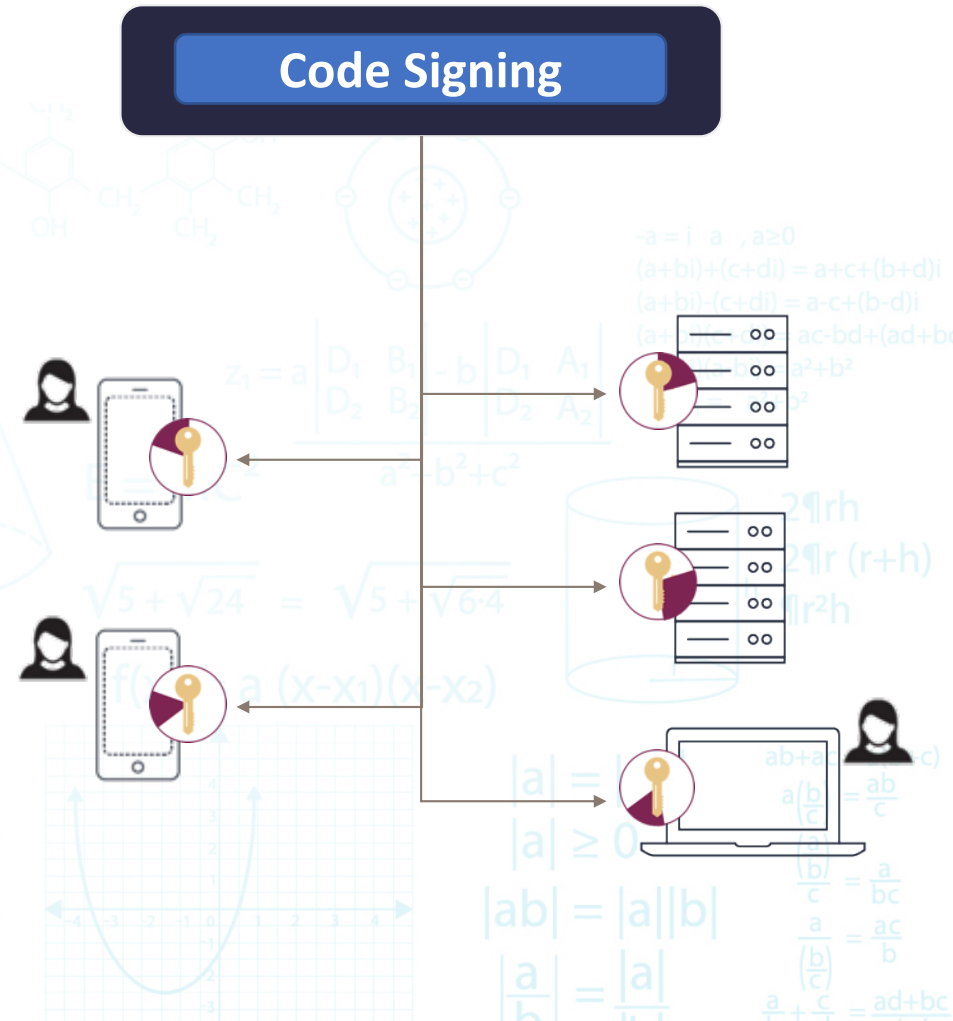
Key material never exists in its entirety at any point of its lifecycle

# Deploying an MPC-Based Virtual HSM



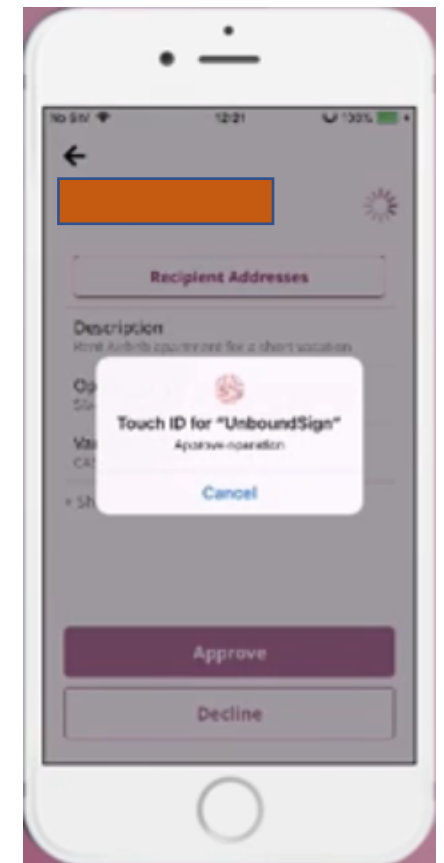
# From Key Theft to Key Misuse

- **Consider a code signing application**
  - A single malicious signing is a complete failure
  - Protection from key theft is not enough
- **Using MPC can define flexible quorums based on multiple sets and arbitrary thresholds**
  - 2 out of the 3 parties at R&D, AND
  - 1 out of the 2 parties at legal
- **Can set quorum sizes depending on need**
- **All parties participate and so approval is cryptographically enforced**



# Two-Factor Authentication with MPC

- **Mobiles are powerful computing devices, but extremely vulnerable**
- **Virtual smartcard / OTP token on mobile**
  - Mobile and server hold key shares and compute via MPC
    - Key never present on mobile at any time
  - Refresh key sharing at *every single operation*
    - Strong anti-cloning and detection
  - All operations are audited at the server as well as mobile
    - Full visibility into operations
  - Easy to use – mobile is always with you
    - This is a big security advantage
  - Easy deployment and management





# Summary

- MPC is a mature technology and ready for deployment
- MPC still requires high expertise to deploy
  - What problems can be solved efficiently?
  - Tailoring protocols to specific needs
  - Subtleties in published protocols (papers almost never specific basic checks)
- MPC is being used in production, and its use and interest are quickly growing

A large group of people, likely employees, are posed in a modern office space. They are arranged in several rows, some sitting on the floor and others standing. The image is overlaid with a semi-transparent blue filter. In the background, a wall features the word "UNBOUND" and the phrase "INNOVATION BEYOND LIMITS" with a geometric logo. The overall atmosphere is professional and collaborative.

# UNBOUND

[ WHERE SECURITY IS KEY ]